New Specification

Rewarding Learning

ADVANCED
General Certificate of Education

Further Mathematics

Assessment Unit A2 1
assessing
Pure Mathematics

[AFM11]

Assessment

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) FURTHER MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

1.		M1	Trying to expand out into separate series
		W1	Correctly
		W3	Substituting $2n$ correctly into each of 3 series
		W1	Simplified to give correct answer
2.	(a)	MW1	Re-writing denominator in terms of u
		M1	Trying to differentiate
		W1	Correct
		W1	Correct integral created (in terms of u)
		W1	Correctly integrated
		W1	Correct final answer
	(b)	M1	Changing to t and limits
		W1	Integration correct
		M1	Using limits
		W1	Correct answer
3.	(i)	MW1	Correct 1st partial fraction
		MW1	Correct 2nd partial fraction
		M1	Trying to equate numerators
		W3	1 mark for each of A, B and C correct
		W1	Correct answer
	(ii)	M1	Integrals identified (allow ft from "their" answer in (i))
		W2	1 mark for each correct term
4.	(i)	M1	Trying to use parts
		W1	Correct u
		MW1	Correct $\frac{\mathrm{d}u}{\mathrm{d}x}$
		W1	Correct $\frac{\mathrm{d}v}{\mathrm{d}x}$ and v
		MW1	Correctly substituted
		W1	Correct answer
	(ii)	M1	Relates I_2 to I_1
		MW1	Relates I_1 to I_0
		W1	Correct I_1 found
		W1	Correct I_2
5.		M1	Tests for $n = 1$
		W1	Shows true for $n = 1$
		M1	Assumes true for $n = k$
		W1	Tries to test for $n = k + 1$
		W1	Correct for $n = k + 1$
		W1	Correct final statement

6. (i) MW1 Rewrites in correct form
   M1 Tries to find IF
   W1 Correct IF found
   MW1 Multiples through by IF
   MW1 Recognise LHS as derivative of $\frac{A}{t}$
   MW1 Correctly integrated
   MW1 Tries to find c
   W1 Correct answer

   (ii) MW1 Correctly substitutes $t = 50$ into their formula
   W1 Correct answer

   (iii) W2 Any 2 valid answers

7. (a) (i) M1 Uses function of a function to differentiate $\sqrt{1 + 4x^2}$
   W1 Correct numerator
   W1 Correct denominator
   M1 Tries to simplify
   W1 Correct answer

   (ii) MW1 Correct 2nd derivative
   MW1 Correct 3rd derivative
   W1 Correct values of functions when $t = 0$
   W1 Correct answer

   (b) M1 Uses correct approximations for sine and cosine
   W1 sin $3x$ correct
   W1 cos $6x$ correct
   MW1 Correct answer

8. (a) M1 Trying to write sinh and cosh in exponential form
   W1 Correct
   W1 1st set of brackets expanded correctly
   W1 2nd set of brackets expanded correctly
   W1 Simplified correctly in exponential format
   W1 Re-written in hyperbolic form correctly

   (b) M1 Trying to write sinh and cosh in exponential form
   W1 Correct
   W1 Re-written as quadratic
   MW1 Recognises disguised quadratic
   W1 Correct solution of quadratic to give 2 solutions for e^x
   W1 Selection of correct solution for e^x
   W1 Correct answer

9.	(i)	M1	Trying to set up auxiliary equation
		W1	Correct equation
		W1	Correct solutions
		M1	Trying to setup CF
		W1	Correct CF
		M1	Trying to setup correct PI
		MW1	Correct 1st and 2nd derivatives
		W1	Correct C
		W1	Correct D
		W1	Correct answer
	(ii)	MW1	Correct value of A
		M1	Trying to differentiate their answer to (i)
		W1	Correct
		W1	Correct value of B
		W1	Correct solution
10.	(a)	M1	Writing x and y in terms of θ
		W1	Obtaining $x^2 + y^2 = r^2$
		W1	Using double angle formula correctly
		W1	Replacing trig functions in terms of x, y and r
		W1	Replacing r^2 in terms of x, y
		W1	Correct answer
	(b)	M1	Trying to use correct formula for area
		W1	Correct limits
		W1	Replacing r^2 in terms of a and θ
		W1	Using $\sec^2\theta = 1 + \tan^2\theta$
		MW1	Trying to integrate
		W1	Both terms correct
		W1	Substituting limits correctly
		W1	Correct answer
11.	(a)	M1	Using $[r, \theta]$ form for $1 + \mathrm{i}$
		W1	Correct r
		W1	Correct θ
		MW1	Use of De Moivre to obtain $(1 + \mathrm{i})^{4n}$ in correct $[r, \theta]$ form
		MW1	Use of De Moivre to obtain $(1 - \mathrm{i})^{4n}$ in correct $[r, \theta]$ form
		W1	Correct $(1 + \mathrm{i})^{4n}$ in trig. form
		W1	Correct $(1 - \mathrm{i})^{4n}$ in trig. form
		W1	Correct answer
	(b) (i)	M1	Trying to use De Moivre
		W1	Correct expansion of $(\cos\theta + \mathrm{i}\sin\theta)^4$
		W1	Separating into real and imaginary parts
		W1	Correct answer for $\cos 4\theta$

(ii) M1 Trying to use $\sin^2\theta + \cos^2\theta = 1$

W1 Correct answer

(iii) MW1 Using $\theta = \dfrac{\pi}{12}$ to give $\cos 4\theta = \dfrac{1}{2}$

MW1 Substituting to obtain quadratic equation

W1 Solving for $\cos^2\theta \rightarrow 2$ solutions

W1 Solving for $\cos\theta \rightarrow 4$ solutions

W1 Elimination of negative solutions

W1 Using $\cos\dfrac{\pi}{12} > \dfrac{1}{2}$

W1 Selection of correct answer

12. (i) M1 Setting $x = \tanh y$

W1 Changing to exponential form

W1 Correct numerator

W1 Correct denominator

W1 Changing to $e^{2y} = \dfrac{1+x}{1-x}$

W1 Changing to y = ... in log form

(ii) M1 Trying to substitute $\sin x$ into part (i) answer

W1 Correctly

MW1 Differentiate both terms correctly

W1 Trying to combine fractions

M1 Trying to re-write in terms of $\cos x$ only

W1 Correct answer

(iii) M1 Trying to use parts

W4 1 mark each for correct u, $\dfrac{dv}{dx}$, $\dfrac{du}{dx}$, v

MW1 Substituted correctly

W1 Correct substitution of limits

W1 Correct answer (allow in decimal form to 3s.f.)

			AVAILABLE MARKS
1	$\sum_{r=1}^{2n}(2r-1)^2 = \sum_{r=1}^{2n}(4r^2 - 4r + 1)$		
	$= \sum_{r=1}^{2n} 4r^2 - \sum_{r=1}^{2n} 4r + \sum_{r=1}^{2n} 1$	M1 W1	
	$= \frac{4}{6}(2n(2n+1)(4n+1)) - 4\left(\frac{2n(2n+1)}{2}\right) + 2n$	W1 W1 W1	
	$= \frac{n}{3}(32n^2 + 24n + 4 - 24n - 12 + 6)$		
	$= \frac{2n}{3}(16n^2 - 1)$	W1	6
2	**(a)** $4x^2 + 4x + 3 = (2x+1)^2 + 2 = u^2 + 2$	MW1	
	$u = 2x + 1 \quad \frac{\mathrm{d}u}{\mathrm{d}x} = 2$	M1 W1	
	$\int \frac{\mathrm{d}x}{\sqrt{4x^2 + 4x + 3}} = \frac{1}{2}\int \frac{\mathrm{d}u}{\sqrt{u^2 + 2}}$	W1	
	$= \frac{1}{2}\ln\left(u + \sqrt{u^2 + 2}\right) + c$	W1	
	$= \frac{1}{2}\ln\left(2x + 1 + \sqrt{4x^2 + 4x + 3}\right) + c$	W1	
	(b) $\int_0^1 \frac{1}{\sqrt{1 - x^2}}\mathrm{d}x = \lim_{t \to 1} \int_0^t \frac{1}{\sqrt{1 - x^2}}\mathrm{d}x$	M1 W1	
	$= \lim_{t \to 1} \left[\sin^{-1} x\right]_0^t = \lim_{t \to 1} \left(\sin^{-1} t\right)$	M1	
	$= \frac{\pi}{2}$	W1	10
3	**(i)** $\frac{2x^2 + x + 7}{(x-1)(x^2+4)} \equiv \frac{A}{(x-1)} + \frac{Bx + C}{(x^2+4)}$	MW1	
	$\equiv \frac{A(x^2+4) + (Bx + C)(x-1)}{(x-1)(x^2+4)}$	MW1	
	$A(x^2+4) + (Bx + C)(x-1) \equiv 2x^2 + x + 7$	M1	
	Let $x = 1$ $\quad 5A = 10 \quad A = 2$	W1	
	Coeff. of x^2 $\quad A + B = 2 \quad B = 0$	W1	
	Number term $\quad 4A - C = 7 \quad C = 1$	W1	
	$\frac{2x^2 + x + 7}{(x-1)(x^2+4)} \equiv \frac{2}{x-1} + \frac{1}{x^2+4}$	W1	
	(ii) $\int \frac{2x^2 + x + 7}{(x-1)(x^2+4)}\mathrm{d}x = \int \frac{2}{x-1}\mathrm{d}x + \int \frac{1}{x^2+4}\mathrm{d}x$	M1	
	$= 2\ln(x-1) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$	W1 W1	10

		Marks	AVAILABLE MARKS
4	**(i)** $\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$	M1	
	$u = (\ln x)^n \qquad \frac{du}{dx} = n(\ln x)^{n-1}\left(\frac{1}{x}\right)$	W1 MW1	
	$\frac{dv}{dx} = 1 \qquad v = x$	W1	
	$I_n = \int(\ln x)^n dx = x(\ln x)^n - \int x\, n(\ln x)^{n-1}\left(\frac{1}{x}\right)dx$	MW1	
	$= x(\ln x)^n - n\int(\ln x)^{n-1}dx$		
	$= x(\ln x)^n - n\, I_{n-1}$	W1	
	(ii) $\left[I_2\right]_1^2 = \int_1^2(\ln x)^2 dx = \left[x(\ln x)^2\right]_1^2 - 2\left[I_1\right]_1^2$	M1	
	$I_1 = x\ln x - I_0 = x\ln x - \int 1\, dx$	MW1	
	$= x\ln x - x$	W1	
	$\left[I_2\right]_1^2 = \left[x(\ln x)^2 - 2(x\ln x - x)\right]_1^2$		
	$= \left[2(\ln 2)^2 - 4\ln 2 + 4\right] - 2$		
	$= 2(\ln 2)^2 - 4\ln 2 + 2$	W1	10
5	Let $n = 1$ LHS $= 3^0 = 1$ RHS $= \frac{3-1}{2} = 1$ $\therefore$ true for $n = 1$	M1 W1	
	Assume true for $n = k$ i.e. $\sum_{r=1}^{k} 3^{r-1} = \frac{3^k - 1}{2}$	M1	
	Let $n = k + 1$ $\sum_{r=1}^{k+1} 3^{r-1} = \sum_{r=1}^{k} 3^{r-1} + 3^k$	W1	
	$= \frac{3^k - 1}{2} + 3^k$		
	$= \frac{3^{k+1} - 1}{2}$	W1	
	True for $n = k \Rightarrow$ true for $n = k + 1$ True for $n = 1 \therefore$ true for all n	W1	6

6 **(i)** $\frac{dA}{dt} - \frac{A}{t} = 1$ MW1

AVAILABLE MARKS

$\int P\mathrm{d}t = \int -\frac{1}{t}\mathrm{d}t = -\ln t = \ln\left(\frac{1}{t}\right)$ M1

$e^{\int P\mathrm{d}t} = e^{\ln\left(\frac{1}{t}\right)} = \frac{1}{t}$ W1

$\therefore \frac{1}{t}\frac{dA}{dt} - \frac{1}{t^2}A = \frac{1}{t}$ MW1

$\frac{d}{dt}\left(\frac{A}{t}\right) = \frac{1}{t}$ MW1

$\therefore \frac{A}{t} = \ln t + c$

$A = t(\ln t + c)$ MW1

$t = 20 \quad A = 300 \qquad \therefore 300 = 20(\ln 20 + c)$ MW1

$c = 15 - \ln 20$

$A = t(\ln t + 15 - \ln 20)$ W1

(ii) $t = 50 \qquad A = 50(\ln 50 + 15 - \ln 20)$ MW1

= 796 million to 3 s.f. W1

(iii) Extrapolation risky, time period very long, banking a volatile business etc.
Any sensible observations. W2 12

AVAILABLE MARKS

7 **(a)** **(i)** $f(x) = \ln\left(2x + \sqrt{1 + 4x^2}\right)$

$$f'(x) = \frac{2 + \frac{1}{2}(1 + 4x^2)^{-\frac{1}{2}} 8x}{2x + (1 + 4x^2)^{\frac{1}{2}}} = \frac{2 + 4x(1 + 4x^2)^{-\frac{1}{2}}}{2x + (1 + 4x^2)^{\frac{1}{2}}}$$ M1 W1 W1

$$= \frac{2(1 + 4x^2)^{\frac{1}{2}} + 4x}{(1 + 4x^2)^{\frac{1}{2}}\left(2x + (1 + 4x^2)^{\frac{1}{2}}\right)}$$ M1

$$= \frac{2}{(1 + 4x^2)^{\frac{1}{2}}}$$ W1

(ii) $$f''(x) = \frac{-8x}{(1 + 4x^2)^{\frac{3}{2}}}$$ MW1

$$f'''(x) = -\left[8(1 + 4x^2)^{-\frac{3}{2}} + 8x\left(-\frac{3}{2}\right)(1 + 4x^2)^{-\frac{5}{2}} 8x\right]$$ MW1

$f(0) = 0 \qquad f'(0) = 2 \qquad f''(0) = 0 \qquad f'''(0) = -8$ W1

$\therefore f(x) = 2x - \frac{4x^3}{3}$ W1

(b) $\sin x \approx x \qquad \cos x \approx 1 - \frac{x^2}{2}$ M1

$\sin 3x \approx 3x \qquad \cos 6x \approx 1 - 18x^2$ W1 W1

$$\lim_{x \to 0}\left(\frac{1 - \cos 6x - 2x\sin 3x}{x^2}\right) = \lim_{x \to 0}\left(\frac{1 - (1 - 18x^2) - 6x^2}{x^2}\right)$$

$$= \lim_{x \to 0}\left(\frac{12x^2}{x^2}\right) = 12$$ MW1 13

8 **(a)** $$\cosh x \cosh y + \sinh x \sinh y = \left(\frac{e^x + e^{-x}}{2}\right)\left(\frac{e^y + e^{-y}}{2}\right) + \left(\frac{e^x - e^{-x}}{2}\right)\left(\frac{e^y - e^{-y}}{2}\right)$$ M1 W1

$$= \frac{1}{4}\left(e^{x+y} + e^{x-y} + e^{-x+y} + e^{-x-y} + e^{x+y} - e^{x-y} - e^{-x+y} + e^{-x-y}\right)$$ W1 W1

$$= \frac{1}{4}\left[2e^{x+y} + 2e^{-x-y}\right] = \frac{e^{x+y} + e^{-(x+y)}}{2}$$ W1

$= \cosh(x + y)$ W1

(b) $4\cosh x + 5\sinh x = 2$

$$4\left(\frac{e^x + e^{-x}}{2}\right) + 5\left(\frac{e^x - e^{-x}}{2}\right) = 2$$ M1 W1

$9e^x - e^{-x} - 4 = 0$

$9e^{2x} - 4e^x - 1 = 0$ W1

$9(e^x)^2 - 4e^x - 1 = 0$ MW1

$$e^x = \frac{4 \pm \sqrt{16 + 36}}{18} = \frac{2 \pm \sqrt{13}}{9}$$ W1

$e^x > 0 \qquad \therefore e^x = \frac{2 + \sqrt{13}}{9}$ W1

$x = \ln\left(\frac{2 + \sqrt{13}}{9}\right)$ W1 13

				AVAILABLE MARKS
9	**(i)**	$m^2 + 2m + 10 = 0$	M1 W1	
		$m = \dfrac{-2 \pm \sqrt{4 - 40}}{2} = -1 \pm 3\mathrm{i}$	W1	
		C.F. $y = \mathrm{e}^{-x}(A\cos 3x + B\sin 3x)$	M1 W1	
		$y = Cx + D \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = C$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$	M1 MW1	
		$20x - 6 = 0 + 2C + 10(Cx + D)$ $10C = 20 \quad \therefore C = 2$	W1	
		$2C + 10D = -6 \quad \therefore D = -1$	W1	
		General Solution $\quad y = \mathrm{e}^{-x}(A\cos 3x + B\sin 3x) + 2x - 1$	W1	
	(ii)	$x = 0 \quad y = 0 \quad \therefore \; 0 = A - 1 \;\therefore\; A = 1$	MW1	
		$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{-x}(-3A\sin 3x + 3B\cos 3x) - \mathrm{e}^{-x}(A\cos 3x + B\sin 3x) + 2$	M1 W1	
		$x = 0 \quad \dfrac{\mathrm{d}y}{\mathrm{d}x} = 3B - A + 2 = 6 \quad \therefore B = \dfrac{5}{3}$	W1	
		$y = \mathrm{e}^{-x}\left(\cos 3x + \dfrac{5}{3}\sin 3x\right) + 2x - 1$	W1	15
10	**(a)**	$x = r\cos\theta \qquad y = r\sin\theta \qquad x^2 + y^2 = r^2$	M1 W1	
		$r = 3\sin 2\theta \qquad \Rightarrow r = 6\sin\theta\cos\theta$	W1	
		$\left(x^2 + y^2\right)^{\frac{1}{2}} = 6.\dfrac{x}{r}.\dfrac{y}{r} = \dfrac{6xy}{r^2} = \dfrac{6xy}{x^2 + y^2}$	W1 W1	
		$\therefore \left(x^2 + y^2\right)^{\frac{3}{2}} = 6xy$ **or** $\left(x^2 + y^2\right)^3 = 36x^2y^2$	W1	
	(b)	Area $= \dfrac{1}{2}\displaystyle\int_0^{\frac{\pi}{4}} r^2\,\mathrm{d}\theta = \dfrac{1}{2}\int_0^{\frac{\pi}{4}} a^2(1 + \tan\theta)^2\,\mathrm{d}\theta$	M1 W1 W1	
		$= \dfrac{a^2}{2}\displaystyle\int_0^{\frac{\pi}{4}} (1 + 2\tan\theta + \tan^2\theta)\,\mathrm{d}\theta = \dfrac{a^2}{2}\int_0^{\frac{\pi}{4}} (2\tan\theta + \sec^2\theta)\,\mathrm{d}\theta$	W1 MW1	
		$= \dfrac{a^2}{2}\left[2\ln\lvert\sec\theta\rvert + \tan\theta\right]_0^{\frac{\pi}{4}}$	W1 W1	
		$= \dfrac{a^2}{2}[\ln 2 + 1]$ square units	W1	14

AVAILABLE MARKS

11 **(a)** $(1+\mathrm{i})^{4n}$ $\quad r=\sqrt{2}$ $\quad \theta=\frac{\pi}{4}$ $\quad \left[\sqrt{2},\frac{\pi}{4}\right]^{4n}$ $\quad =\left[4^n,n\pi\right]$ M1 W2

$(1-\mathrm{i})^{4n}$ $\quad r=\sqrt{2}$ $\quad \theta=-\frac{\pi}{4}$ $\quad \left[\sqrt{2},-\frac{\pi}{4}\right]^{4n}$ $\quad =\left[4^n,-n\pi\right]$ MW2

$(1+\mathrm{i})^{4n}-(1-\mathrm{i})^{4n}=4^n(\cos n\pi+\mathrm{i}\sin n\pi)-4^n\left(\cos(-n\pi)+\mathrm{i}\sin(-n\pi)\right)$ W1
W1

$=4^n(2\mathrm{i}\sin n\pi)$
$=0$ W1

(b) **(i)**
$(\cos\theta+\mathrm{i}\sin\theta)^4$
$=\cos^4\theta+4\mathrm{i}\cos^3\theta\sin\theta+6\mathrm{i}^2\cos^2\theta\sin^2\theta+4\mathrm{i}^3\cos\theta\sin^3\theta+\mathrm{i}^4\sin^4\theta$ M1
$=\cos^4\theta-6\sin^2\theta\cos^2\theta+\sin^4\theta+\mathrm{i}(4\sin\theta\cos^3\theta-4\sin^3\theta\cos\theta)$ W1 W1
$(\cos\theta+\mathrm{i}\sin\theta)^4=\cos4\theta+\mathrm{i}\sin4\theta$
$\Rightarrow\cos4\theta=\cos^4\theta-6\sin^2\theta\cos^2\theta+\sin^4\theta$ W1

(ii) $\cos4\theta=\cos^4\theta-6\cos^2\theta(1-\cos^2\theta)+(1-\cos^2\theta)^2$ M1

$=8\cos^4\theta-8\cos^2\theta+1$ W1

(iii) $\theta=\frac{\pi}{12}$ $\quad 4\theta=\frac{\pi}{3}$ $\quad \cos4\theta=\frac{1}{2}$ MW1

Let $u=\cos^2\theta$

$\therefore\frac{1}{2}=8u^2-8u+1\quad\Rightarrow\quad16u^2-16u+1=0$ MW1

$u=\dfrac{16\pm\sqrt{16^2-4\times16}}{32}=\dfrac{2\pm\sqrt{3}}{4}$ W1

$\cos\theta=\pm\sqrt{\dfrac{2\pm\sqrt{3}}{4}}=\dfrac{\pm\sqrt{2\pm\sqrt{3}}}{2}$ W1

Let $\theta=\frac{\pi}{12}$ $\quad \cos\frac{\pi}{12}>0\quad\therefore\quad\cos\frac{\pi}{12}=\dfrac{\sqrt{2\pm\sqrt{3}}}{2}$ W1

Consider $\dfrac{\sqrt{2-\sqrt{3}}}{2}<\frac{1}{2}$ W1

But $\cos\frac{\pi}{3}=\frac{1}{2}$

$\frac{\pi}{12}<\frac{\pi}{3}\quad\therefore\ \cos\frac{\pi}{12}>\cos\frac{\pi}{3}\qquad\therefore\cos\frac{\pi}{12}\neq\dfrac{\sqrt{2-\sqrt{3}}}{2}$

$\therefore\cos\frac{\pi}{12}=\dfrac{\sqrt{2+\sqrt{3}}}{2}$ W1 21

		AVAILABLE MARKS
12 **(i)** $y = \tanh^{-1} x \qquad \tanh y = x = \dfrac{e^{2y} - 1}{e^{2y} + 1}$	M1 W1 W2	
$x(e^{2y} + 1) = e^{2y} - 1 \quad \Rightarrow \quad e^{2y} = \dfrac{1 + x}{1 - x}$	W1	
$\therefore 2y = \ln\left(\dfrac{1 + x}{1 - x}\right)$		
$y = \tanh^{-1} x = \frac{1}{2}\ln\left(\dfrac{1 + x}{1 - x}\right)$	W1	
(ii) $\tanh^{-1}(\sin x) = \frac{1}{2}\ln\left(\dfrac{1 + \sin x}{1 - \sin x}\right) = \frac{1}{2}(\ln(1 + \sin x) - \ln(1 - \sin x))$	M1 W1	
$\dfrac{d}{dx}[\tanh^{-1}(\sin x)] = \frac{1}{2}\left[\dfrac{\cos x}{1 + \sin x} - \dfrac{-\cos x}{1 - \sin x}\right]$	MW1	
$= \dfrac{\cos x}{2}\left[\dfrac{2}{1 - \sin^2 x}\right]$	W1	
$= \dfrac{1}{\cos x}$	M1	
$= \sec x$	W1	
(iii) $\displaystyle\int_0^{\frac{\pi}{6}} \sec^2 x \tanh^{-1}(\sin x)\,dx \qquad u = \tanh^{-1}(\sin x) \qquad \dfrac{du}{dx} = \sec x$	M1 W1 W1	
$\dfrac{dv}{dx} = \sec^2 x \qquad v = \tan x$	W1 W1	
$= \left[\tanh^{-1}(\sin x)\tan x\right]_0^{\frac{\pi}{6}} - \displaystyle\int_0^{\frac{\pi}{6}} \tan x \sec x\,dx$	MW1	
$= \left[\tanh^{-1}(\sin x)\tan x\right]_0^{\frac{\pi}{6}} - [\sec x]_0^{\frac{\pi}{6}}$		
$= \left[\tanh^{-1}\left(\frac{1}{2}\right)\tan\frac{\pi}{6}\right] - [0] - \left[\sec\frac{\pi}{6}\right] + 1$	W1	
$= \dfrac{1}{2\sqrt{3}}\ln 3 - \dfrac{2\sqrt{3}}{3} + 1$	W1	20
	Total	**150**